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IMPEDANCE MATCHING BY RE-ENTRANT STUB LINE

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ABSTRACT. This article describes a new method of matching a load to a transmission line. This is suitable for narrow band matching when the load V.S.W.R. is greater than 4.

INTRODUCTION

It is well known that for efficient transfer of power and for other considerations, a transmission line is required to be matched to the load. In many cases, however, the load presents an impedance different from the characteristic impedance of the transmission line necessitating some device to match the load to the line. At frequencies where lumped networks are not suitable, the devices normally employed for narrow band matching are stub lines, quarter-wave transformers, or dielectric sleeves. To these may be added a re-entrant stub line matching section as shown in Fig. 1.

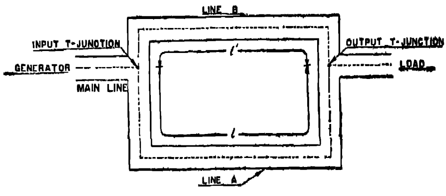

Fig. 1. Re-entrant stub line matching section using coaxial line.

Two transmission lines A and B are connected in parallel to form a re-entrant double stub line. It will be shown that if the two line lengths l and l' are appropriately selected the impedance at the output T-junction is transformed to the desired value at the input junction. The range of impedances at the output T-junction that can be matched to the input line depends upon the characteristic impedances of lines A and B relative to the main line. Theoretically, by proper choice of the characteristic impedances of lines A and B, it is possible to make the main line flat for any load V.S.W.R. at the output junction. In practice, however, the case of importance will be when the characteristic impedance of *all* the lines are identical. Only this case will, therefore, be considered. This consideration restricts the use of the re-entrant stub line section as a matching device to load V.S.W.R. equal to or higher than 4. The analysis together with some experimental results are presented in this article.

MATRIX PARAMETERS OF TRANSMISSION LINES

The input and output currents I_1 and I_2 of a uniform, loss-less transmission line of length l shown in Fig. 2(a) can be expressed in terms of the corresponding voltages E_1 and E_2 by

$$\left.\begin{aligned} I_1 &= -j\,Y_0(\cot\beta l)E_1 + j\,Y_0(\operatorname{cosec}\beta l)E_2 \\ I_2 &= j\,Y_0(\operatorname{cosec}\beta l)E_1 - jY_0(\cot\beta l)E_2 \end{aligned}\right\} \quad \ldots \quad (1)$$

where Y_0 and β ($= 2\pi/\lambda$) are the characteristic admittance and the phase constant respectively of the line and λ is the wavelength.

In matrix notation Equ. (1) is written as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -jY_0\cot\beta l & jY_0\operatorname{cosec}\beta l \\ jY_0\operatorname{cosec}\beta l & -jY_0\cot\beta l \end{bmatrix} \times \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \quad \ldots \quad (2)$$

$$\text{i.e.,} \qquad [\,I\,] = [\,Y\,] \times [\,E\,]$$

We can, therefore, represent a given length of transmission line by a two-terminal pair netwsork, shown in Fig. 2(b), provided their admittance matrices

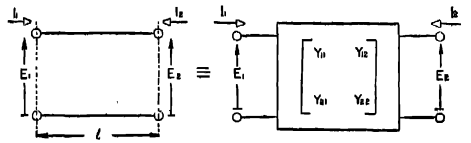

Fig. 2(a). Loss-loss transmission line of length l and characteristic admittance Y_0.

Fig. 2(b). Equivalent network representation of the transmission line.

are identical. For the equivalent network the elements of the admittance are obtained from

$$[\,Y\,] = jY_0 \begin{bmatrix} -\cot\beta l & \operatorname{cosec}\beta l \\ \operatorname{cosec}\beta l & -\cot\beta l \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$\text{i.e.,} \qquad \left.\begin{aligned} Y_{11} = Y_{22} &= -jY_0\cot\beta l \\ Y_{12} = Y_{21} &= jY_0\operatorname{cosec}\beta l \end{aligned}\right\} \quad \ldots \quad (3)$$

Let us now consider the two lines A and B, one (A) of length l and the other (B) of length l', each of characteristic admittance Y_0 connected as shown in Fig. 1. The equivalent network representation is shown in Fig. 3 where the unprimed quantities refer to line A and the primed quantities to line B.

The resultant [Y] matrix of the parallel combination of the two networks is thus from Equ. (3).

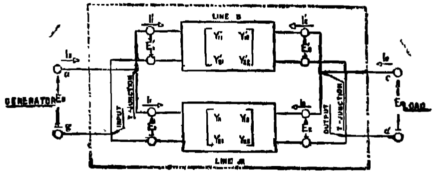

Fig. 3. Equivalent network representation of the re-entrant stub lin

$$\text{Resultant} \quad [\,Y\,] = \begin{bmatrix} Y_{11}+Y_{11}' & Y_{12}+Y'_{12} \\ Y_{21}+Y'_{21} & Y_{22}+Y'_{22} \end{bmatrix}$$

$$= jY_0 \begin{bmatrix} -(\cot\beta l+\cot\beta l') & (\operatorname{cosec}\beta l+\operatorname{cosec}\beta l') \\ (\operatorname{cosec}\beta l+\operatorname{cosec}\beta l') & -(\cot\beta l+\cot\beta l') \end{bmatrix} \qquad (4)$$

The matrix representation (4) now leads us to a single equivalent network of the re-entrant stub line section as far as the input (terminals $a-b$) and output (terminals $c-d$) are concerned. This is shown in Fig. 4.

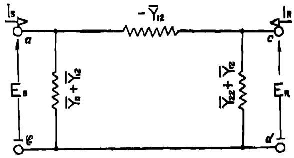

Fig. 4. Resultant network configuration of the re-entrant stub line.

In the above representation,

$$\left.\begin{aligned} \bar{Y}_{11} &= Y_{11}+Y'_{11} = \bar{Y}_{22} = Y_{22}+Y'_{22} \\ &= -jY_0(\cot\beta l+\cot\beta l'); \\ \bar{Y}_{12} &= (Y_{12}+Y'_{12}) = (Y_{21}+Y'_{21}) \\ &= jY_0(\operatorname{cosec}\beta l+\operatorname{cosec}\beta l') \end{aligned}\right\} \quad \dots \quad (5)$$

If the value of the load admittance at $c-d$ is $\bar{Y}_R = -(I_R/E_R)$, it is easy to show that the input admittance at $a-b$ looking into the terminals is

$$Y_{in} = \bar{Y}_{11} - [\,\bar{Y}^2_{12}/(\bar{Y}_{22}+Y_R)] \quad \dots \quad (6)$$

IMPEDANCE MATCHING

We now propose to show that for Y_R lying within a certain range defined later, it is possible to make $Y_{in} = Y_0$ by appropriate choice of the lengths of the lines A and B. The condition required for matching is $Y_{in} = Y_0 + j(0)$. The desired line lengths $\theta(=\beta l)$ and $\theta'(=\beta l')$ are then obtained from Eq (5) and (6) and are governed by the following two simaltaneous equations :

$$\left.\begin{aligned} g_R(\operatorname{cosec}\theta+\operatorname{cosec}\theta')^2 &= g_R^2+(b_R+\cot\theta+\cot\theta')^2 \\ \frac{(b_R+\cot\theta+\cot\theta')(\operatorname{cosec}\theta+\operatorname{cosec}\theta')^2}{g_R^2+(b_R+\cot\theta+\cot\theta')^2} &= \cot\theta+\cot\theta' \end{aligned}\right\} \quad \ldots (7)$$

where $$g_R - jb_R = Y_R/Y_0 = y_R.$$

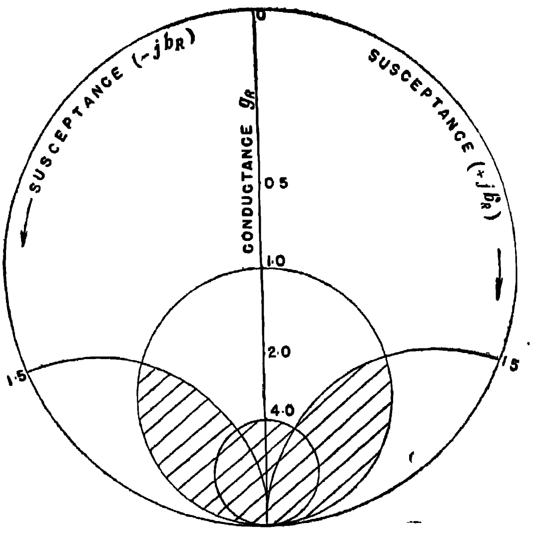

Fig. 5. Smith chart showing the range of impedance (shaded region) that can be matched.

Equ. (7) on simplification yields

$$\left.\begin{aligned} \cot\theta+\cot\theta' &= a \\ \operatorname{cosec}\theta+\operatorname{cosec}\theta' &= \pm b \end{aligned}\right\} \quad \ldots (8)$$

where
$$a = b_R/(g_R-1)$$
$$b = \sqrt{g_R(1+a^2)}$$

The solutions of Equ. (8) are :

$$\left.\begin{aligned} &\text{(i)} \quad \cot\frac{\theta}{2} = \frac{1}{2}\,(d_1 \pm \sqrt{d_1^2 - 4c_1}), \quad \cot\frac{\theta'}{2} = \frac{1}{2}\,(d_1 \mp \sqrt{d_1^2 - 4c_1} \\ &\text{(ii)} \quad \cot\frac{\theta}{2} = \frac{1}{2}\,(d_2 \pm \sqrt{d_2^2 - 4c_2}), \quad \cot\frac{\theta'}{2} = \frac{1}{2}\,(d_2 \mp \sqrt{d_2^2 - 4c_2}) \end{aligned}\right\} \quad \ldots \quad (9)$$

where
$$c_1 = (b+a)/(b-a), \quad c_2 = 1/c_1$$
$$d_1 = a+b, \qquad d_2 = a-b.$$

It can be easily shown that solutions for θ and θ' may also be obtained from the alternative forms of Eq. (8) given below in Eq. (10) and (11) :

$$\operatorname{cosec}^2\theta \mp b \operatorname{cosec}\theta + \left(\frac{a^2}{b^2-a^2} + \frac{b^2-a^2}{4} \right) = 0 \quad \ldots \quad (10)$$

and an identical equation for cosec θ'.

$$\cot^2\theta - a\cot\theta + \left(\frac{b^2}{b^2-a^2} - \frac{b^2-a^2}{4} \right) = 0 \quad \ldots \quad (11)$$

and an identical equation for $\cot\theta'$.

Physically realisable solutions for θ and θ' are obtained provided the inequality $g_R + b_R^2/(g_R-1) \geqslant 4$ is satisfied. The range of y_R where it is possible to get an impedance match is thus shown by the shaded region in the Smith chart (Fig. 5). It would be seen that whenever load V.S.W.R. $\geqslant 4$, an impedance match at the input T-junction can always be obtained by locating the putput T-junction at any appropriate point inside the shaded region.

A similar analysis for the case when the characteristic admittances of lines A and B are each half the input and output lines, shows that all load V.S.W.R's can be matched.

V. S. W. R. IN EACH LINE

The normalised values of the output admittances y_{A0} and y_{B0} terminating the lines A and B respectively (see Fig. 3), are dependent upon the line lengths and the load admittance and are given by

$$
\begin{aligned}
y_{A0} &= -(I_2/E_2) \\
&= \frac{y_{12}(y_R+y'_{22})-y_{22}y'_{12}}{y_{12}+y'_{12}} \\
&= \frac{y_R \operatorname{cosec}\theta - j\operatorname{cosec}\theta\cot\theta' + j\cot\theta\operatorname{cosec}\theta'}{\operatorname{cosec}\theta+\operatorname{cosec}\theta'} \qquad \ldots (12)
\end{aligned}
$$

$$
\begin{aligned}
y_{B0} &= -(I'_2/E_2') \\
&= \frac{y_{12}'(y_R+y_{22})-y_{22}'y_{12}}{y_{12}+y_{12}'} \\
&= \frac{y_R \operatorname{cosec}\theta' - j\operatorname{cosec}\theta'\cot\theta + j\cot\theta'\operatorname{cosec}\theta}{\operatorname{cosec}\theta+\operatorname{cosec}\theta'} \qquad \ldots (13)
\end{aligned}
$$

In the above all admittances denoted by lower case symbols are normalised with respect to Y_0.

Once the complex values of y_{A0} and y_{B0} are obtained from Egns. (12) and (13), the reflection coefficients Γ_{A0} and Γ_{B0} at the output ends of lines A and B respectively are readily determined. Thus

$$
\left.\begin{aligned}
\Gamma_{A0} &= (1-y_{A0})/(1+y_{A0}) \\
\Gamma_{B0} &= (1-y_{B0})/(1+y_{B0})
\end{aligned}\right\} \qquad \ldots (14)
$$

The V.S.W.R.'s set up in each line are then :

$$
\left.\begin{aligned}
\rho_A &= (1+|\Gamma_{A0}|)/(1-|\Gamma_{A0}|) \quad \text{for line } A \\
\rho_B &= (1+|\Gamma_{B0}|)/(1-|\Gamma_{B0}|) \quad \text{for line } B
\end{aligned}\right\} \qquad \ldots (15)
$$

SPECIAL CASE OF RESISTIVE LOADS ($g_R \geqslant 4$)

Let us consider the case when the load V.S.W.R. $\geqslant 4$. If now the load presented at the output T-junction is resistive, $b_R = 0$ and $y_R = g_R(\geqslant 4)$. For such a situation, assumed to be brought about by proper location of the matching device along the load side of the transmission line, Eqns. (8), (12) to (15) are very much simplified and are respectively given below :

$$
\begin{aligned}
&\cot\theta+\cot\theta' = 0 \\
&\operatorname{cosec}\theta+\operatorname{cosec}\theta' = \pm b \\
&\text{whence } \theta+\theta' = \pi \\
&\text{and } \operatorname{cosec}\theta = \pm b/2
\end{aligned}
$$

$$\left.\begin{aligned} y_{A0} &= (g_R/2)+j\cot\theta \\ y_{B0} &= (g_R/2)-j\cot\theta \end{aligned}\right\}$$

$$\Gamma_{A0} = \Gamma_{B0} = [(g_R-3)/(g_R+5)]^{\frac{1}{2}}$$

$$\rho_A = \rho_B = (\sqrt{g_R+5}+\sqrt{g_R-3})/(\sqrt{g_R+5}-\sqrt{g_R-3})$$

The last one shows that when the load V.S.W.R. $>> 5$, ρ_A and ρ_B are each approximately half the load V.S.W.*R*.

EXPERIMENTAL VERIFICATION

Three re-entrant line matching sections were made with two *T*-junctions and lengths of rigid air dielectric coaxial lines (outer 5/8″ OD × 1/32″ wall tubing and inner 0.244″ dia) of characteristic impedance 50 ohms having the following electrical lengths :

(i) $l = 26.34$ cm, $l' = 99.90$ cm
(ii) $l = 16.34$ cm, $l' = 89.90$ cm.
(iii) $l = 15.04$ cm, $l' = 89.90$ cm

Calculated performances of each of these when the output *T*-junction is placed at a voltage minimum (resistive load) show that the input should be matched at

(i) $\lambda = 252.48$ cm, frequency $= 118.82$ Mc/s
when load V.S.W.R. $= 10.75$, ile., $g_R = 10.75$

(ii) $\lambda = 212.48$ cm, frequency $= 141.19$ Mc/s
when load V.S.W.R. $= 18.51$, i.e., $g_R = 18.51$

(iii) $\lambda = 209.88$ cm, frequency $= 142.94$ Mc/s
When load V.S.W.R. $= 21.13$, i.e., $g_R = 21.13$

Measurement of admittance at the input T-junction with an U.H.F. Admittance Meter (General Radio Co., type 1602B) showed prefect agreement in each case within the accuracy of the instrument.

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